

Package ‘GRS.test’

October 12, 2022

Type Package

Title GRS Test for Portfolio Efficiency, Its Statistical Power
Analysis, and Optimal Significance Level Calculation

Version 1.2

Date 2022-06-29

Author Jae H. Kim <jaekim8080@gmail.com>

Maintainer Jae H. Kim <jaekim8080@gmail.com>

Description Computational resources for test proposed by Gibbons, Ross, Shanken (1989)<[DOI:10.2307/1913625](https://doi.org/10.2307/1913625)>.

It also has the functions for the power analysis and the choice of the optimal level of significance. The optimal level is determined by minimizing the expected loss from hypothesis testing.

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NeedsCompilation no

Repository CRAN

Date/Publication 2022-07-01 21:30:21 UTC

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GRS.test-package *GRS Test for Portfolio Efficiency, Its Statistical Power Analysis, and Optimal Significance Level Calculation*

Description

Computational resources for test proposed by Gibbons, Ross, Shanken (1989)<DOI:10.2307/1913625>. It also has the functions for the power analysis and the choice of the optimal level of significance. The optimal level is determined by minimizing the expected loss from hypothesis testing.

Details

The DESCRIPTION file:

```
Package:      GRS.test
Type:        Package
Title:       GRS Test for Portfolio Efficiency, Its Statistical Power Analysis, and Optimal Significance Level Calculation
Version:     1.2
Date:       2022-06-29
Author:      Jae H. Kim <jaekim8080@gmail.com>
Maintainer:  Jae H. Kim <jaekim8080@gmail.com>
Description: Computational resources for test proposed by Gibbons, Ross, Shanken (1989)<DOI:10.2307/1913625>. It also
License:     GPL-2
```

Index of help topics:

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GRS.MLtest      GRS Test Statistic and p-value based on Maximum
                Likelihood Estimator for Covariance matrix
GRS.Power       Statistical Power of the GRS test
GRS.Powerfunc   Power functions for the GRS test
GRS.T           Sample Size Selection for the GRS test
GRS.optimal     Optimal Level of Significance for the GRS test:
                Normality Assumption
GRS.optimalboot Optimal Level of Significance for the GRS test:
                Bootstrapping
GRS.optimalbootweight Weighted Optimal Level of Significance for the
                GRS test: Bootstrapping
GRS.optimalweight Weighted Optimal Level of Significance for the
                GRS test: Normality Assumption
GRS.test        GRS test and Model Estimation Results
GRS.test-package GRS Test for Portfolio Efficiency, Its
                Statistical Power Analysis, and Optimal
                Significance Level Calculation
data           Fama-French Data: 25 size-B/M portfolio and
                risk factors, obtained from French's library
```

The package accompanies the working paper:

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

The function `GRS.test` returns the GRS test statistics with model estimation results.

The function `GRS.MLtest` provides an alternative test statistic with θ and θ^* estimation results.

Additional functions for the power analysis and calculation of optimal level of significance are also included.

Author(s)

Jae H. Kim <jaekim8080@gmail.com>

Maintainer: Jae H. Kim <jaekim8080@gmail.com>

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica*, 57,1121-1152. <DOI:10.2307/1913625>

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, *Journal of Financial Economics*, 1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

See Also

The examples replicate the results reported in Fama and French (1993) and Kim and Shamsuddin (2016)

Examples

```
data(data)
factor.mat = data[1:342,2:4]           # Fama-French 3-factor model
ret.mat = data[1:342,8:ncol(data)]    # 25 size-BM portfolio returns
GRS.test(ret.mat,factor.mat)$GRS.stat # Table 9C of Fama-French (1993)
```

data *Fama-French Data: 25 size-B/M portfolio and risk factors, obtained from French's library*

Description

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Usage

```
data("data")
```

Format

A data frame with 630 observations on the following 32 variables.

date date

RM_RF Market Excess Return

SMB SMB

HML HML

RMW RMW

CMA CMA

MOM MOM

P11 Portfolio Returns

P12 Portfolio Returns

P13 Portfolio Returns

P14 Portfolio Returns

P15 Portfolio Returns

P21 Portfolio Returns

P22 Portfolio Returns

P23 Portfolio Returns

P24 Portfolio Returns

P25 Portfolio Returns

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P43 Portfolio Returns

P44 Portfolio Returns

P45 Portfolio Returns

P51 Portfolio Returns

P52 Portfolio Returns

P53 Portfolio Returns

P54 Portfolio Returns

P55 Portfolio Returns

Details

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Source

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
Monthly from 1963 to 2015

References

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>
Fama and French, 2015, A five-factor asset-pricing model, *Journal of Financial Economics*, 116-1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

Examples

```
data(data)
y=ts(data[,2], frequency=12, start=c(1950,1))
plot.ts(y)
```

GRS.MLtest	<i>GRS Test Statistic and p-value based on Maximum Likelihood Estimator for Covariance matrix</i>
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Description

W statistic given in (7) of GRS (1989) <DOI:10.2307/1913625>

Usage

```
GRS.MLtest(ret.mat, factor.mat)
```

Arguments

ret.mat	portfolio return matrix, T by N
factor.mat	matrix of risk factors, T by K

Details

T: sample size, N: number of portfolio returns, K: number of risk factors

Value

GRS.stat	GRS test statistic
GRS.pval	its p-value
theta	maximum Sharpe ratio of the K factor portfolios
thetas	slope of the efficient frontier based on all assets
ratio	theta/thetas, proportion of the potential efficiency

Note

Applicable to CAPM as well as a multi-factor model

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica*, 57,1121-1152. <DOI:10.2307/1913625>

See Also

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, *Journal of Financial Economics*, 116-1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

Examples

```
data(data)
factor.mat = data[1:342,2:4]           # Fama-French 3-factor model
ret.mat = data[1:342,8:ncol(data)]    # 25 size-BM portfolio returns
GRS.MLtest(ret.mat,factor.mat)       # See column (iv), Table 9C of Fama-French (1993)
```

GRS.optimal

Optimal Level of Significance for the GRS test: Normality Assumption

Description

The optimal level is calculated by minimizing expected loss from hypothesis testing

The F-distributions are used to calculate the power, under the normality assumption

Usage

```
GRS.optimal(T, N, K, theta, ratio, p = 0.5, k = 1, Graph = TRUE)
```

Arguments

T	sample size
N	the number of portfolio returns
K	the number of risk factors
theta	maximum Sharpe ratio of the K factor portfolios
ratio	theta/thetas, proportion of the potential efficiency

p	prior probability for H0, default is p = 0.5
k	relative loss, k = L2/L1, default is k = 1
Graph	show graph if TRUE. No graph otherwise

Details

Based on the power calculation of the GRS test, as in GRS (1989) <DOI:10.2307/1913625>.

The blue square in the plot is the point where the expected loss is minimized.

The red horizontal line in the plot indicates the point of the conventional level of significance (alpha = 0.05).

Value

opt.sig	Optimal level of significance
opt.crit	Critical value corresponding to opt.sig
opt.beta	Type II error probability corresponding to opt.sig

Note

ratio = theta/thetas

thetas = maximum Sharpe ratio of the K factor portfolios: GRS (1989) <DOI:10.2307/1913625>

Author(s)

Jae H. Kim

References

Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

GRS.optimal(T=90, N=25, K=3, theta=0.25, ratio=0.4) # Figure 3 of Kim and Shamsuddin (2017)

GRS.optimalboot

*Optimal Level of Significance for the GRS test: Bootstrapping***Description**

The optimal level is calculated by minimizing expected loss from hypothesis testing

The bootstrap is used to calculate the power.

The power is calculated at the estimated values (unrestricted) of parameters under H1.

Usage

```
GRS.optimalboot(ret.mat, factor.mat, p=0.5, k=1, nboot=3000, wild=FALSE, Graph=TRUE)
```

Arguments

ret.mat	portfolio return matrix, T by N
factor.mat	matrix of risk factors, T by K
p	prior probability for H0, default is $p = 0.5$
k	relative loss, $k = L2/L1$, default is $k = 1$
nboot	the number of bootstrap iterations, the default is 3000
wild	if TRUE, wild bootstrap is conducted; if FALSE (default), bootstrap is based on iid residual resampling
Graph	show graph if TRUE (default). No graph otherwise

Details

The blue square in the plot is the point where the expected loss is minimized.

The red horizontal line in the plot indicates the point of the conventional level of significance ($\alpha = 0.05$).

The function also returns the density functions under H0 and H1 (black and red curves, with vertical line the critical value at the optimal level).

Value

opt.sig	Optimal level of significance
opt.crit	Critical value corresponding to opt.sig
opt.beta	Type II error probability corresponding to opt.sig

Note

The example below sets nboot=500 for faster execution, but a higher number is recommended.

Author(s)

Jae H. Kim

References

- Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.
- Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>
- Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>
- Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

- Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

```
data(data)
n=60; m1=nrow(data)-n+1; m2=nrow(data) # Choose the last n observations from the data set
factor.mat = data[m1:m2,2:6]           # Fama-French 5-factors
ret.mat = data[m1:m2,8:ncol(data)]     # 25 size-BM portfolio returns
GRS.optimalboot(ret.mat,factor.mat,p=0.5,k=1,nboot=500,wild=TRUE,Graph=TRUE)
```

GRS.optimalbootweight *Weighted Optimal Level of Significance for the GRS test: Bootstrapping*

Description

The optimal level is calculated by minimizing expected loss from hypothesis testing

The bootstrap is used to calculate the power.

The non-centrality parameter is estimated and its bootstrap distribution is obtained.

9 percentiles from this distribution is used to calculate the power and the optimal level.

These optimal levels are weighted using the weights from the density of the bootstrap distribution of lambda.

Usage

```
GRS.optimalbootweight(ret.mat,factor.mat,p=0.5,k=1,nboot=3000,wild=FALSE,Graph=TRUE)
```

Arguments

ret.mat	portfolio return matrix, T by N
factor.mat	matrix of risk factors, T by K
p	prior probability for H0, default is $p = 0.5$
k	relative loss, $k = L2/L1$, default is $k = 1$
nboot	the number of bootstrap iterations, the default is 3000
wild	if TRUE, wild bootstrap is conducted; if FALSE (default), bootstrap is based on iid residual resampling
Graph	show graph if TRUE (default). No graph otherwise

Details

Power is calculated based on the bootstrap

The plot shows the bootstrap distribution of lambda (non-centrality parameter)

See Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Value

opt.sig	Optimal level of significance
opt.crit	Critical value corresponding to opt.sig

Note

The example below sets nboot=500 for faster execution, but a higher number is recommended.

Author(s)

Jae H. Kim

References

- Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.
- Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:<http://dx.doi.org/10.1016/j.jempfin.2015.08.006>>
- Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>
- Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

```

data(data)
n=60; m1=nrow(data)-n+1; m2=nrow(data) # Choose the last n observations from the data set
factor.mat = data[m1:m2,2:6]           # Fama-French 5-factors
ret.mat = data[m1:m2,8:ncol(data)]     # 25 size-BM portfolio returns
GRS.optimalboot(ret.mat, factor.mat, p=0.5, k=1, nboot=500, wild=TRUE, Graph=TRUE)

```

GRS.optimalweight	<i>Weighted Optimal Level of Significance for the GRS test: Normality Assumption</i>
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Description

The optimal level is calculated by minimizing expected loss from hypothesis testing.

The F-distributions are used to calculate the power, under the normality assumption

The power is calculated using a range of non-centrality parameters (λ), following a folded-normal distribution.

The weights are obtained from the density function of folded-normal distribution.

See, for details, Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach.

Usage

```
GRS.optimalweight(T, N, K, theta, ratio, delta = 3, p = 0.5, k = 1, Graph = TRUE)
```

Arguments

T	sample size
N	the number of portfolio returns
K	the number of risk factors
theta	maximum Sharpe ratio of the K factor portfolios
ratio	theta/thetas, proportion of the potential efficiency
delta	the standard deviation of the folded-normal distribution, default is 3
p	prior probability for H0, default is p = 0.5
k	relative loss, $k = L2/L1$, default is $k = 1$
Graph	show graph if TRUE. No graph otherwise

Details

Based on the power calculation of the GRS test, as in GRS (1989) <DOI:10.2307/1913625>.

The plot shows the folded-normal distribution.

Value

opt.sig Optimal level of significance
 opt.crit Critical value corresponding to opt.sig

Note

ratio = theta/thetas

thetas = maximum Sharpe ratio of the K factor portfolios: GRS (1989) <DOI:10.2307/1913625>

Author(s)

Jae H. Kim

References

Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

GRS.optimalweight(T=90, N=25, K=3, theta=0.25, ratio=0.4)

GRS.Power

Statistical Power of the GRS test

Description

Calculates the power of the GRS test with density functions under H0 and H1

Usage

GRS.Power(T, N, K, theta, ratio, alpha = 0.05, xmax = 10, Graph = "TRUE")

Arguments

T	sample size
N	the number of portfolio returns
K	the number of risk factors
theta	maximum Sharpe ratio of the K factor portfolios
ratio	theta/thetas, proportion of the potential efficiency
alpha	the level of significance, default is 0.05
xmax	the support of the density is from 0 to xmax, default is 10
Graph	show graph if TRUE. No graph otherwise

Details

Calculate the power following GRS (1989) <DOI:10.2307/1913625>

The distribution under H1 is based on the value of theta and ratio

Under H0: ratio = 1; under H1: ratio < 1

Value

Power	power of the test
Critical.value	critical value at alpha

Note

The graph option plots the density functions of the GRS test under H0 and H1.

The blue vertical line represents the critical value at alpha level of significance

The black density function is the one under H0, and the gray-shaded area is level of significance.

The red one is the one under H1, and the red-shaded area is the power.

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica*, 57,1121-1152. <DOI:10.2307/1913625>

See Also

GRS(1989) <DOI:10.2307/1913625>

Examples

GRS.Power(T=120, N=25, K=3, theta=0.3, ratio=0.5) # Figure 1 of Kim and Shamsuddin (2016)

GRS.Powerfunc

Power functions for the GRS test

Description

The function plots the power functions for a range of sample size (T), given the other parameter values

Usage

```
GRS.Powerfunc(Tvec, N, K, theta, alpha = 0.05)
```

Arguments

Tvec	a vector of sample sizes
N	the number of portfolio returns
K	the number of risk factors
theta	maximum Sharpe ratio of the K factor portfolios
alpha	the level of significance, default is 0.05

Details

The power is plotted against the ratio= θ/θ_s , the proportion of potential efficiency

Value

Power: Matrix of power values plotted

Note

Under H_0 : ratio = 1, so the power = alpha when ratio = 1.

The power increases as the ratio declines from 1.

The power increases with sample size, so the upper power function is associated with larger sample size.

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica*, 57,1121-1152. <DOI:10.2307/1913625>

See Also

GRS (1989) <DOI:10.2307/1913625>

Examples

```
GRS.Powerfunc(Tvec=c(60,120),N=25, K=3,theta=0.3) # Figure 2 of Kim and Shamsuddin (2016)
```

GRS.T *Sample Size Selection for the GRS test*

Description

Given the desired level of Type I and II error probabilities, the function returns the sample size required.

Usage

```
GRS.T(N, K, theta, ratio, alpha, beta, Tmax = 10000)
```

Arguments

N	the number of portfolio returns
K	the number of risk factors
theta	maximum Sharpe ratio of the K factor portfolios
ratio	theta/thetas, proportion of the potential efficiency
alpha	the desired level of significance, or Type I error probability
beta	the desired level of Type II error probability
Tmax	the maximum number of sample size, default is 10000

Details

the desired level of power = 1 - beta

Value

Required.T	required sample size
Critical.value	the corresponding critical value

Note

Critical.value is from the F-distribution with $df1=N$ and $df2=Required.T-N-K$ degrees of freedom, at the alpha level of significance.

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica*, 57,1121-1152. <DOI:10.2307/1913625>

See Also

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

Examples

```
GRS.T(N=25,K=3,theta=0.25,ratio=0.4,alpha=0.05, beta=0.05, Tmax=5000)
```

GRS.test

GRS test and Model Estimation Results

Description

Wu statistic given in (5) of GRS (1989) <DOI:10.2307/1913625>

The function also provide estimation results for asset pricing models

Usage

```
GRS.test(ret.mat, factor.mat)
```

Arguments

ret.mat	portfolio return matrix, T by N
factor.mat	matrix of risk factors, T by K

Details

T: sample size, N: number of portfolio returns, K: number of risk factors

Value

GRS.stat	GRS test statistic
GRS.pval	its p-value
coef	matrix of coefficient estimates from N equations, N by (K+1)
resid	matrix of residuals from N equations, T by N
tstat	matrix of t-statistics for coefficients, N by (K+1)
se	matrix of standard errors for coefficients, N by (K+1)
R2	matrix of R-squares for N equations, N by 1

Note

Applicable to CAPM as well as a multi-factor model

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, *Econometrica*, 57,1121-1152. <DOI:10.2307/1913625>

See Also

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, *Journal of Financial Economics*, 1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

Examples

```
data(data)
factor.mat = data[1:342,2:4]           # Fama-French 3-factor model
ret.mat = data[1:342,8:ncol(data)]    # 25 size-BM portfolio returns
GRS.test(ret.mat,factor.mat)$GRS.stat # See Table 9C of Fama-French (1993)
```

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